STAT 410 - Linear Regression

Lecture 9

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Assumptions for linear regression models

- Recall the linear regression model with k regressors:

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i \tag{1}$$

- Assumptions:
  1. The error term has zero mean, or equivalently, $\mathrm{E}(y_i) = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik}$.
  2. The error term has constant variance.
  3. The errors are uncorrelated.
  4. $\varepsilon_i \sim$ Normal.

Residual analysis

- We shall examine the model adequacy by analyzing **residuals**.
- The residual is $e_i = y_i - \hat{y}_i$, or $\mathbf{e} = (\mathbf{I} - \mathbf{H})\mathbf{y}$ in a matrix form.
- Standardized residuals to remove the scale:

$$s_i = \frac{e_i}{\hat{\sigma}} = \frac{e_i}{\sqrt{MS_{Res}}}$$

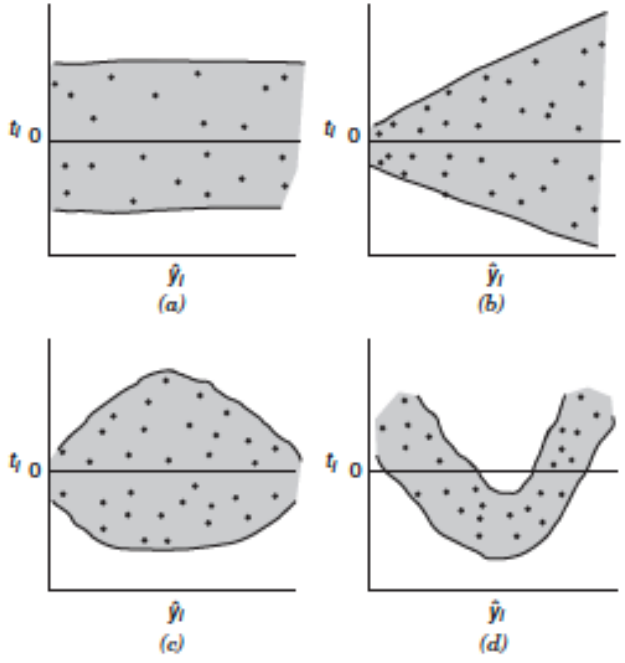
- The plain residual and its plots are useful for checking the model assumptions:
  - QQ plot
  - Residual vs. fitted values
  - Residual vs. regressors

QQ plot - the normal assumption

- A quantile-quantile normal plot, or simply QQ plot, plots sample quantiles vs. theoretical quantiles of a standard normal.
- In the ideal case where a sample is i.i.d. from a normal distribution, we expect to see a straight line in its QQ plot.
- QQ plots may help diagnose heavy/light tailed or skewed error distributions.
- Possible solutions to violated normal assumption:
  - to cite George Box's quote
  - robust regression
  - transformation of response
- It is sometimes well behaved even if the errors are not normal.

Residuals vs. fitted values

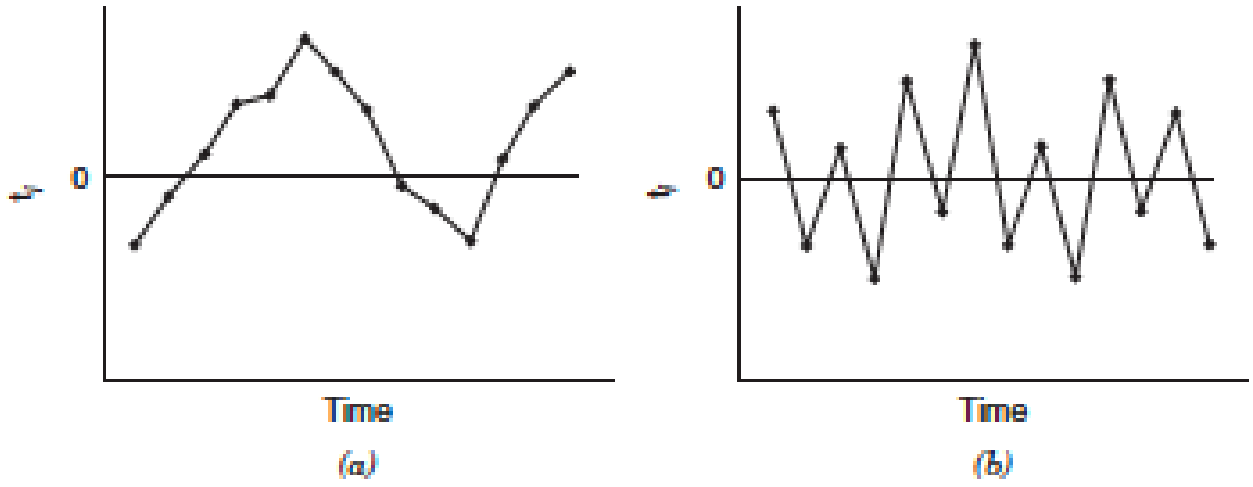
- We expect to see a random scatter of points around the horizontal axis.
- This is because: under MLR, $\mathbf{e}'\hat{\mathbf{y}} = 0$; under the normal assumption, $\mathbf{e}$ and $\hat{\mathbf{y}}$ are independent.



- (a) Satisfactory; (b) and (c) Heterogeneous variances; (d) Nonlinearity
- Possible solutions: transformation of response/regressor, adding polynomial terms, etc.

Residuals vs. regressors

- It is satisfactory to have a horizontal band containing the residuals without any clear pattern.
- For example, the plots below exhibit a pattern of autocorrelation.



- (a) Positively correlated errors; (b) Negatively correlated errors.
- Possible solutions: to build a time series model that specifically addresses the autocorrelation.